A Damage-Dependent Finite Element Analysis for Fiber-Reinforced Composite Laminates

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Summary

A progressive damage methodology has been developed to predict damage growth and residual strength of fiber-reinforced composite structure with through penetrations such as a slit. The methodology consists of a damage-dependent constitutive relationship based on continuum damage mechanics. Damage is modeled using volume averaged strain-like quantities known as internal state variables and is represented in the equilibrium equations as damage induced force vectors instead of the usual degradation and modification of the global stiffness matrix.

Introduction

The characterization of damage in notched composite laminates has led to the development of progressive damage models to predict damage initiation, damage growth, and residual strength. Allen and Harris developed a damage-dependent constitutive model [1,2] which utilizes kinematic-based volume averaged damage variables to represent the effects of matrix cracking and fiber fracture. This model has a matrix crack growth law for fatigue as well as monotonic tension and the kinematic effects of delaminations are modeled empirically. An experimental verification of the accuracy of the model to predict stiffness loss in toughened composite systems was documented by Coats [3]. The progressive damage analysis algorithms of the Allen-Harris progressive damage methodology were implemented into a general purpose finite element code developed by NASA, the Computational Structural Mechanics Testbed (COMET) [4]. The methodology was developed [5] to model mode I and II matrix cracking and tensile fiber fracture in composite center-notch panels that were loaded monotonically in tension. The numerical details of accounting for damage and calculating internal state variables are presented in this paper. The paper will conclude with a comparison of analytical predictions with experimental results for AS4/938 center-notch tension panels.

Progressive Damage Analysis

The progressive damage analysis consists of a constitutive model where internal state variables (ISV) represent the average effects of local deformation due to the various modes of microcrack damage. The constitutive model for an individual ply within a finite element may be written as [1]

$$\sigma_{ij_L} = Q_{ijkl} \left\{ \varepsilon_{kl} - \alpha_{kl} \right\}_L \tag{1}$$

where σ_{ij_L} are the locally averaged components of stress, Q_{ijkl} are the elastic moduli in ply coordinates, and ε_{kl_L} are the locally averaged components of strain. The internal state variables, α_{kl_L}, represent the local deformation effects of the various modes of ply-level damage. The damage dependent constitutive equations (1) can be transformed into laminate equations and substituted into the laminate resultant force and moment equations. This results in damage-dependent laminate resultant force and moment equations and can be written as [4,6]

$$\{N\} = [A]\{\varepsilon_L^o\} + [B]\{\kappa_L\} + \{f^d\} \tag{2}$$

$$\{M\} = [B]\{\varepsilon_L^o\} + [D]\{\kappa_L\} + \{g^d\} \tag{3}$$

where $\{N\}$ and $\{M\}$ are the resultant force and moment vectors, respectively; [A], [B], and [D] are the laminate extensional, coupling, and bending stiffness matrices, respectively; $\{\varepsilon_L^o\}$ is the midplane strain vector; $\{\kappa_L\}$ is the midplane curvature vector; and $\{f^d\}$ and $\{g^d\}$ are the damage resultant force and moment vectors, respectively [4,7]. The application of $\{f^d\}$ and $\{g^d\}$ to the undamaged material will produce midplane strain and curvature contributions equivalent to those resulting from the damage-induced compliance increase. The damage resultant forces and moments are written as:

$$\{f^d\} = -\sum_{k=1}^{n} [\bar{Q}]_k (z_k - z_{k-1}) \{\alpha_L\}_k \tag{4}$$

$$\{g^d\} = -\frac{1}{2}\sum_{k=1}^{n} [\bar{Q}]_k (z_k^2 - z_{k-1}^2) \{\alpha_L\}_k \tag{5}$$

where $[\bar{Q}]_k$ is the transformed elastic modulus matrix for the *k*th ply in laminate coordinates. When equations (2) and (3) are substituted into the plate equilibrium equations, the result is a set of governing differential equations which can be integrated against variations in the displacement components to produce a weak form of the damage-dependent laminated-plate equilibrium equations [4,6]. By substituting displacement interpolation functions into the weak form of the plate equilibrium equations and following well known finite element procedures, the assembled equilibrium equations are obtained as:

$$[K]\{\delta\} = \{F_A\} + \{F_D\} \tag{6}$$

where [K] is the global stiffness matrix of the undamaged structure, $\{\delta\}$ is the global displacement vector, and $\{F_A\}$ is the applied force vector. The damage induced force vector, $\{F_D\}$, is calculated from the element shape functions and the damage resultant forces and moments. Since the effects of damage are represented as damage-induced force vectors on the right-hand side of equation (6), the element stiffness matrix does not need to be recalculated as damage progresses as long as the nonlinearity in the load-deflection curve is not large.

The model requires only the standard ply stiffness and strength properties determined from unidirectional laminates for characterizing the material and predicting residual strength. The progression of damage is predicted by an iterative and incremental procedure outlined in the flow chart shown in Figure 1. This entire progressive failure analysis scheme has been implemented into the finite element formulation in the NASA Computational Mechanics Testbed (COMET) [4] computer code. The first block of Figure 1 is a description of the finite element model. Block numbers are shown in the right hand corner of each box in the flowchart. Blocks 2 and 3 are processors that calculate the element stiffness matrices and assemble and factor the global stiffness matrix. The compliance changes due to damage are accounted for by combining the damage induced force vector with the applied force vector and solving for the global displacements in equation (6). This solution process occurs in blocks 4 and 5 and then the element stress resultants are computed in block 6.

The ply-level strains and stresses are computed in block 7 and as long as the strains in a material element (local volume element or finite element) are less than the critical strains, $\varepsilon_{kl_{crit}}$, no damage exists and the internal state variables have zero values. When the strains reach their critical value, the element is damaged and this damage is represented by an internal state variable. The formulation for α_{kl} was derived based on the assumption that when fiber fracture, mode I (opening mode) matrix cracking, or mode II (shear mode) matrix cracking occur in a ply within an element, the longitudinal, shear, and transverse stresses for that ply in that element are

$$\sigma_{11} = \gamma S_{cr}^{x} \tag{7}$$

$$\sigma_{22} = \beta S_{cr}^{y} \tag{8}$$

$$\sigma_{12} = \psi S_{cr}^{xy} , \tag{9}$$

where S_{cr}^{x}, S_{cr}^{y}, and S_{cr}^{xy} are the ply longitudinal, transverse, and shear critical strengths, respectively; γ, β, and ψ are scale factors that describe the load carrying capability of the material after the occurrence of tensile fiber fracture, mode I matrix cracking, and mode II matrix cracking, respectively. This is illustrated in Figure 2 where it was assumed that for mode I matrix cracking and tensile fiber fracture, 90% of the failure load in the damaged ply of the damaged element would be redistributed to adjacent plies and elements. This was accomplished by setting γ and β to 10%. The load redistribution for mode II matrix cracking was modeled

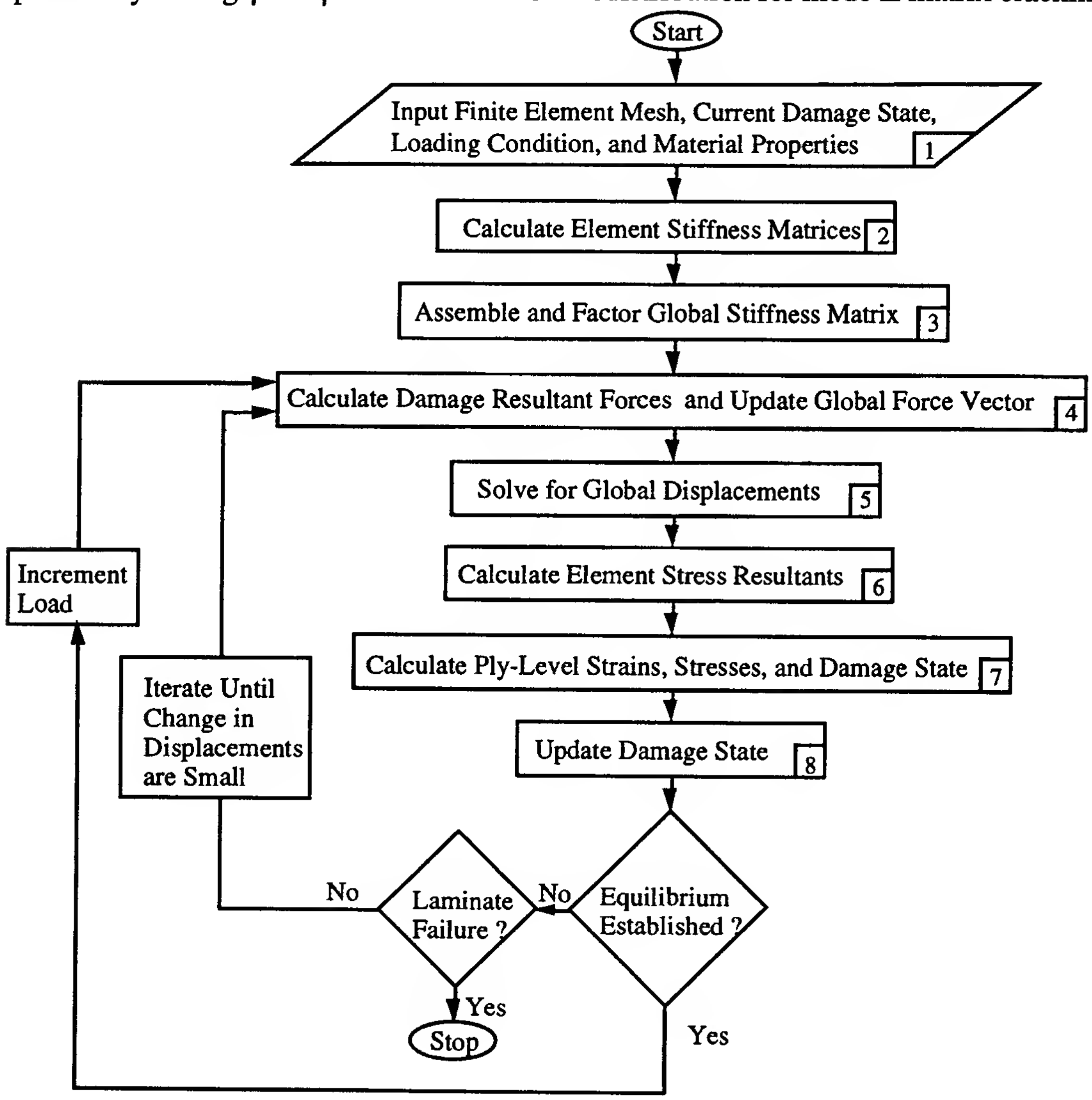

Figure 1. Progressive Failure Analysis.

after Iosipescu shear test data [5, 8].

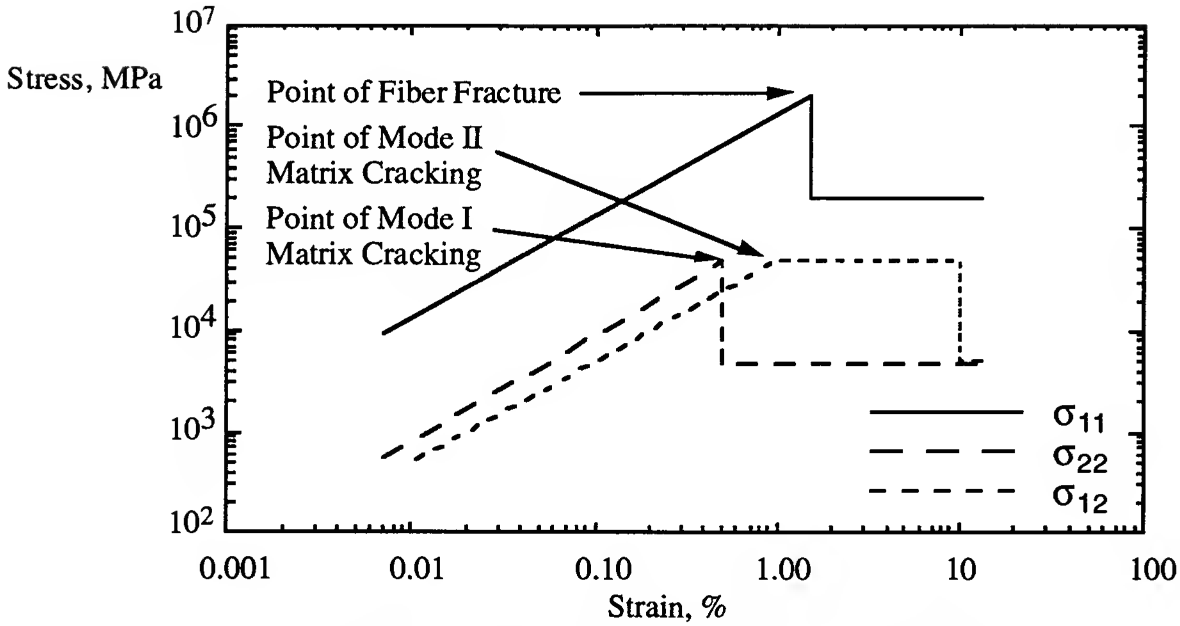

Figure 2. Load Carrying Capability After Ply Failure.

The formulation for α_{kl} was obtained by substituting equations (7), (8), and (9) into the damage-dependent constitutive equations (1) and applying the following equation

$$\alpha_{kl} = \alpha_{kl}^{old} + d\alpha_{kl} \tag{10}$$

where α_{kl} is the updated internal state variable and α_{kl}^{old} is the internal state variable of the previous damage state. The resulting equations are

$$d\alpha_{11} = \begin{cases} \varepsilon_{11} - \alpha_{11}^{old} + \dfrac{1}{Q_{11}Q_{22}-Q_{12}^2}\left(Q_{12}\beta S_{cr}^{y} - Q_{22}\gamma S_{cr}^{x}\right) & \text{if } \alpha_{22}>0; \\ \varepsilon_{11} - \alpha_{11}^{old} + \dfrac{Q_{12}}{Q_{11}}\varepsilon_{22} - \dfrac{\gamma S_{cr}^{x}}{Q_{11}} & \text{if } \alpha_{22}=0 \end{cases} \tag{11}$$

$$d\alpha_{22} = \begin{cases} \varepsilon_{22} - \alpha_{22}^{old} + \dfrac{1}{Q_{11}Q_{22}-Q_{12}^2}\left(Q_{12}\gamma S_{cr}^{x} - Q_{11}\beta S_{cr}^{y}\right) & \text{if } \alpha_{11}>0; \\ \varepsilon_{22} - \alpha_{22}^{old} + \dfrac{Q_{12}}{Q_{22}}\varepsilon_{11} - \dfrac{\beta S_{cr}^{y}}{Q_{22}} & \text{if } \alpha_{11}=0 \end{cases} \tag{12}$$

$$d\alpha_{12} = \frac{-\psi S_{cr}^{xy}}{Q_{66}} + \gamma_{12} - \alpha_{12}^{old} \tag{13}$$

where $d\alpha_{11}$, $d\alpha_{22}$, and $d\alpha_{12}$ are the incremental changes in the internal state variable for tensile fiber fracture, mode I matrix cracking, and mode II matrix cracking, respectively; ε_{11}, ε_{22}, and γ_{12} are the longitudinal, transverse and shear strains, respectively; and Q is the ply-level elastic modulus in ply coordinates. The damage state is updated in block 8 using equation (10) and the ply-level stresses and strains are post-

processed. Note that damaged elements are not removed. Instead, the damage state in the damaged elements increases linearly with strain.

The equilibrated solution is obtained when the damage-induced force vector and the change in displacements become negligible. If the solution shows that equilibrium is established, then the next load increment is applied. If equilibrium is not established, then iteration is performed holding the loads at the current level. As the applied load approaches the catastrophic failure load, the number of iterations required to establish equilibrium increases. This is because the strains in the surrounding undamaged elements approach critical values. In the process of iteration, displacement increases lead to increases in strain levels in these elements and the strains may exceed the failure strains. Therefore, damage may grow from one iteration to the next. Eventually, as loads increase, a load level will be reached where equilibrium is unattainable. This value of load is defined to be the catastrophic failure load, or residual strength.

Results

Center-notch tension panels, as shown in Figure 3, were loaded monotonically to failure. Linear elastic fracture mechanics (LEFM) predictions are compared to the progressive damage model predictions and the experimental data in Figure 3 for four notch sizes and three panel widths. The experimental data is displayed with error bars denoting the maximum and minimum values of the measured strength data. The LEFM prediction, which is plotted as the solid line, was obtained by determining the fracture toughness of the laminate from the residual strength of the smallest specimen with a (2a/W) ratio equal to 1/4. The experimental values for the 22.9 cm notch are about 45% higher than the LEFM based predictions, whereas the progressive damage model correctly predicts this increase in the fracture strength. From these results it is obvious that the finite element model predictions are more accurate than LEFM for wide panels where fracture resistance effects are dominant. Mesh refinement issues concerning the center-notch configuration are discussed in detail in reference [5].

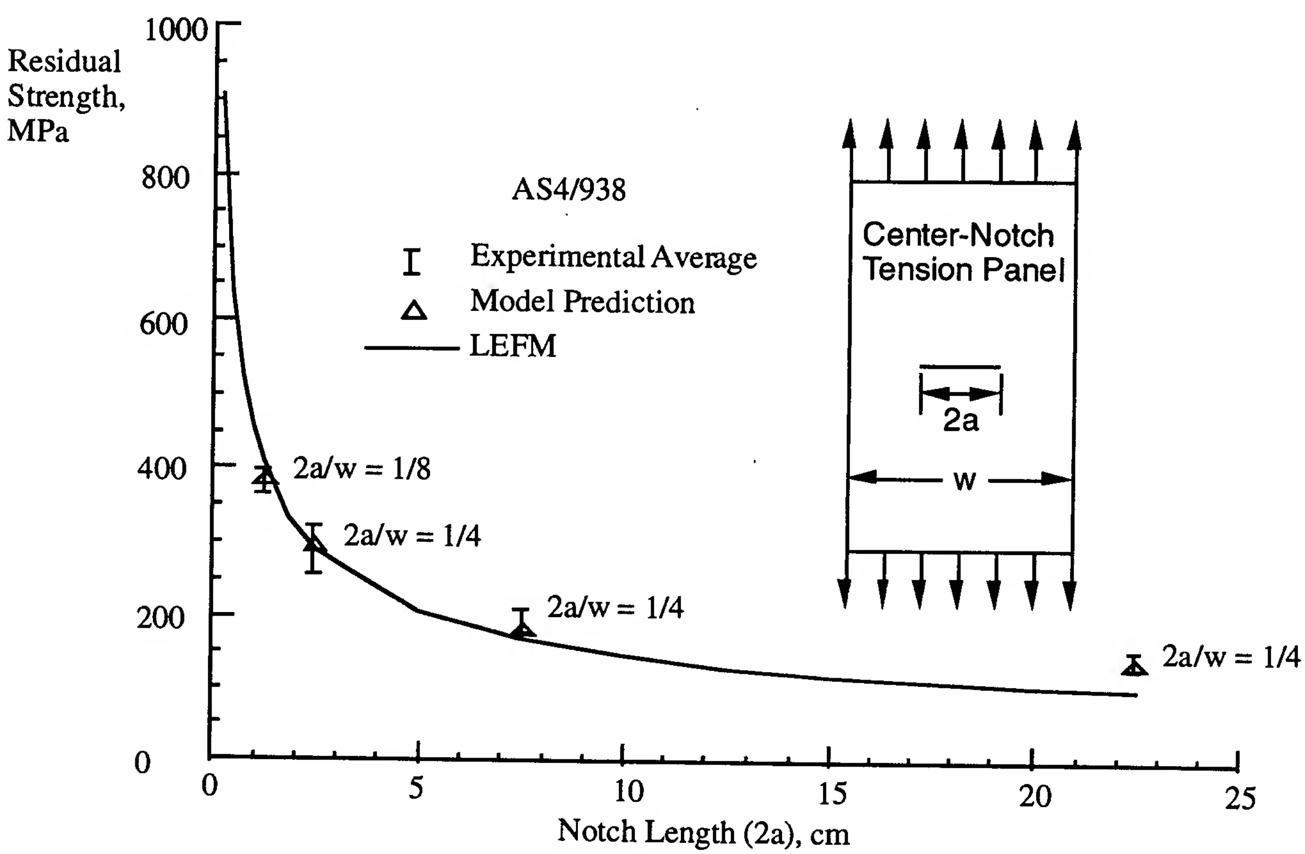

Figure 3. Comparison of LEFM and Model Prediction for the AS4/938 Center-Notch Tension Panels.

Concluding Remarks

A progressive damage methodology has been developed to predict damage growth and residual strength of fiber-reinforced composite structure with through penetrations such as a slit. The methodology consists of a damage-dependent constitutive relationship based on continuum damage mechanics. Damage is modeled using volume averaged strain-like quantities known as internal state variables. The progressive damage analysis algorithms have been implemented into a general purpose finite element code developed by NASA, the **Co**mputational Structural **Me**chanics **T**estbed (COMET). Strain failure criteria are used for damage initiation and the internal state variables are used to numerically simulate damage growth. Damage is represented in the equilibrium equations as damage induced force vectors instead of the usual degradation and modification of the global stiffness matrix. Load is applied incrementally and a classical iterative method is used to establish equilibrium. Analytical predictions of residual strength were made for composite panels with central through slits and the computational results agree within ± 10% of the experimental results.

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